Research on the cultivation strategy of high school mathematical logical reasoning literacy under the new curriculum standard: an example of "proportional series"

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Keywords: logical reasoning literacy; curriculum standards; teaching strategies; proportional sequence; mind map

Abstract: Logical reasoning literacy is one of the six core competencies in mathematics. This article explores the teaching reform elements of logical reasoning literacy in the "High School Mathematics Curriculum Standards (2017 Edition, 2020 Revision)", proposes certain cultivation strategies, and conducts lesson case studies using the "proportional sequence" course content as a carrier, integrating expression communication and mind mapping throughout teaching.

1. Introduction

Since the 21st century, China's mathematics education has undergone significant transformation, gaining unprecedented focus[1]. Quality education aims to develop capable, humane, wise, and innovative individuals, integrating deductive and inductive thinking as foundational elements[2]. The "Implementation Suggestions" in the "General High School Mathematics Curriculum Standards (2017 Edition, Revised 2020)" advocate for a teaching approach centered on students' core mathematical literacy, integrating this cultivation throughout teaching activities[3]. Professor Shi Ningzhong emphasizes the importance of "abstraction, reasoning, and modeling"[4]. Since 2017, logical reasoning literacy has gained prominence, especially in high school research, with the new college entrance exam highlighting core mathematics competencies and logical reasoning as critical skills in middle school mathematics[5]. Improving logical reasoning is essential, as its abstract presentation in textbooks often intimidates students and leads to resistance.

2. Analysis of content

For logical reasoning literacy to take hold, grasping its essence, exploring components, and clarifying core elements of mathematical logical reasoning in middle school teaching is crucial[6].

"Logic" stems from the Greek "λoros" (logos), referring to thought, speech, reason, and rules. Zhou Jing defines reasoning ability as the psychological characteristics enabling understanding of mathematical activities through rational reasoning, formulating hypotheses, and proving them via deductive reasoning while effectively communicating problem-solving processes[7]. Professor Zhang Dianzhou views mathematical logical reasoning as deriving propositions based on logical rules[1]. The new curriculum emphasizes "logical reasoning" as the skill to deduce propositions using rules, involving reasoning from specific to general (induction, analogy) and from general to specific (deduction). Thus, logical reasoning is a vital cognitive approach. It represents not just ability but also a quality of possessing strong thinking attributes. The competency in logical reasoning is an essential skill developed by students in learning mathematics, focusing on the discovery and proficient proof of mathematical propositions.

  DOI: 10.25236/icfmhss.2024.004

3. The essential elements of the educational reform on logical reasoning competence in the new curriculum standard

Comparing the handling of logical reasoning literacy in two versions of the curriculum standards and understanding the changes can provide better guidance for cultivating high school students' logical reasoning literacy[8].

3.1 Emphasize thinking, focus on expertise

Logical reasoning literacy, as an important force in promoting students' ability to think about the world with mathematical thinking, is a new height that students must possess[9].

The comparison of the "Experimental Version of the Ordinary High School Mathematics Curriculum Standards" with the new standards highlights significant differences in keyword frequency. The term "proof process" appears once in the experimental version, rising to eight in the new standards, indicating increased emphasis. Furthermore, "logic" is preferred over "reasoning," showcasing a stronger focus on cognitive awareness of knowledge development principles. This transition from "knowledge-oriented" and "ability-oriented" to "value-driven, competency-focused, anchored in knowledge" emphasizes the importance of developing both mathematical skills and logical reasoning abilities[10].

3.2 Valuing the ability to communicate mathematically

The new curriculum standards have notably raised the usage of "expression" and "communication" to 2.87, 2.8, and 1.49 times the 2003 standards. "Expression" describes attributes verbally, while "communication" conveys ideas via language, tone, expressions, and gestures. The updated standards stress verbal and cognitive expressions in math and detail varying requirements for "expression and communication" in "logical reasoning" (see Table 1).

Table 1. The requirement of "expression and communication"

Proficiency level	Expressions and communications	Notes
Level 1	Being able to clarify the connotation of the discussed issues and express viewpoints in an organized manner during the communication process.	the level that students should achieve after graduating from high school
Level 2	Being able to consistently focus on the theme, have clear viewpoints, and provide logical and well founded arguments during the communication process.	the level that students should reach when taking the college entrance examination
Level 3	Ability to use mathematical language and thinking in a reasonable manner for interdisciplinary expression and communication.	the level that excellent high school graduates can achieve

4. Reflections on Strategies for Cultivating Students' Logical Reasoning Ability

Logic reasoning may seem simple at first glance, but it is actually a complex issue involving both mathematics and philosophy.

4.1 Creating problem scenarios that are close to reality

Effective mathematics teaching focuses on real-life contexts that inspire student thinking. Students often question the relevance of advanced mathematics, believing it unnecessary for everyday purchases. However, buying involves fundamental mathematical skills, while making cost-effective choices demands logical reasoning. Educators should design classroom scenarios linked to real life, guiding students in logical reasoning to appreciate the value of mathematics. Situational learning bridges logical reasoning with real-world problems, underscoring the significance of mathematics in daily life.

4.2 Building a mind map to improve the logical framework

Logical reasoning demands that students integrate the logical relationships among concepts,

propositions, and theorems, focusing on the interconnected integration of knowledge. This necessitates the development of a web-like knowledge system. The mind map serves as an effective tool for constructing such a web-like knowledge system, as it can visualize the thought processes within students. To organize teaching with a mind map, one must commence with various aspects such as introducing new lessons, exploring new concepts, understanding them, and expanding knowledge, ensuring the "mind map" permeates the entirety of classroom instruction.

4.3 Communicating for feedback to enhance quality

The enhancement of mathematical thinking quality is a complex, ongoing process that requires systematic study, utilizing mathematical expressions through knowledge structure development and problem-solving[11]. A classroom atmosphere that promotes "daring to express" is vital, encouraging thoughtful students to share and supporting those who struggle with critical thinking to participate in discussions. Expression and communication should be central to the classroom experience.

At the same time, the teacher guides students to clarify the logical process, distilling general problem-solving pathways and considerations to deepen understanding and foster competence[12].

5. Research on the "proportional sequence" lesson case based on logical reasoning literacy

Using the "geometric sequence" as an example, I will explore the cultivation of logical reasoning skills in teaching, aligning with new curriculum standards and reform elements, by breaking down the study of geometric sequences into four instructional stages[13].

5.1 Build contexts and introduce new knowledge

Question 1: What is the definition and general formula of the difference series?

Scenario: (1)The teacher utilizes multimedia to present diagrams of cell division, illustrating the patterns of cell division that form a sequence: 2, 4, 8, 16...;

(2)Extracting a sequence from the phrase 'half of a foot's coffin, half of it in the day, inexhaustible for eternity': 1, $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{16}$, …;

(3) The formula for compound interest in savings is: Total = Principal × (1 + Rate) Duration. Assuming I deposit 10,000 yuan as New Year money in the bank with an annual interest rate of 5%, what will the total amount be at the end of each year over 5 years?

Design Intent: In reviewing the previous arithmetic sequence lesson, teacher aims to connect it with geometric sequences. By integrating biology and cell division, teacher helps students understand the ratio patterns. Using relatable examples like "lucky money" allows them to see the relevance of mathematics in real life and identify contextual math problems.

Question 2: Observe the first five terms of the above three sequences based on the previous life examples, mathematical formulas, and analogous series, and think about the comparison between each item and the previous term from the second term.

Teacher-Student activity: The instructor guides students to replace "difference" with "ratio," encouraging independent exploration of the series' value law through division.

Design Intent: Using arithmetic sequence insights, teacher have built a framework for geometric sequences, improving analogical understanding of their definitions and properties. Teacher motivates students to express geometric patterns precisely in math language.

Teacher-Student Activity: The teacher presents the "Proportional Sequence" topic, outlines the knowledge structure, and at the course's end, students will fully complete Figure 1.

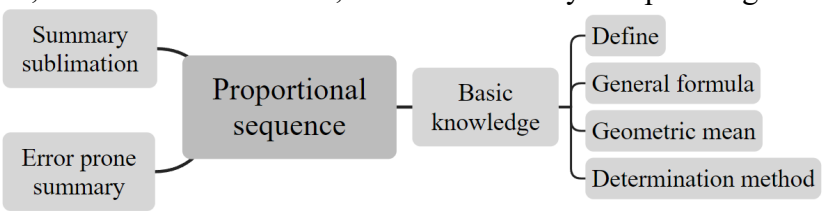

Fig. 1 Equal ratio sequence mind map (unfilled version)

Design Intent: Students will grasp the basic structure and logical sequence of the proportional sequence knowledge in this lesson as a whole, and in the following lessons, they will clarify the connection between mathematical knowledge points through the application and drawing of mind maps[11,12].

5.2 Induction and analogy to explore new knowledge

Teacher-Student Activity: The teacher presents an external knowledge structure diagram of an equal sequence of numbers in Figure 2, inspiring students to make analogies.

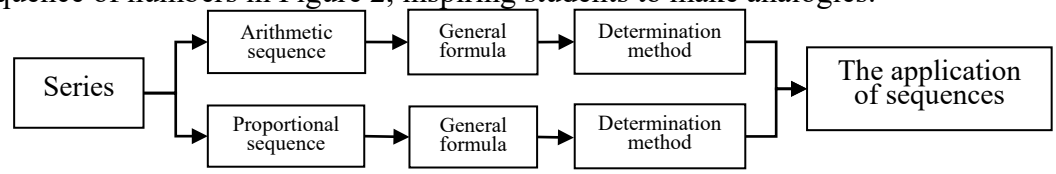

Fig. 2 External knowledge structure diagram of proportional sequence

Question 3: Can you summarize the definition, general term formula, and properties of arithmetic sequences by comparing them to arithmetic sequences?

Teacher-Student Activity: The teacher guides students to draw parallels between the definition, general term formula, and a series of properties of arithmetic sequences, resulting in the definition, general term formula, and properties of geometric sequences.

Question 4: Can you determine the monotonicity of a proportional sequence by analogy to an exponential function?

Teacher-Student Activity: The teacher facilitates student analogies with exponential functions and leads a classification discussion on the monotonicity of geometric sequences.

Design Intention: By examining geometric sequences through their definitions, formulas, properties like the geometric mean, and the monotonicity via exponential functions, students can pinpoint key study areas. This process enhances their ability to translate natural language into symbolic language, improving their mathematical communication skills[14].

Teacher-Student Activity: Collaborate between teachers and students to summarize and organize the connections between geometric sequences and arithmetic sequences, clearly presenting them using a mind map in Figure 3.

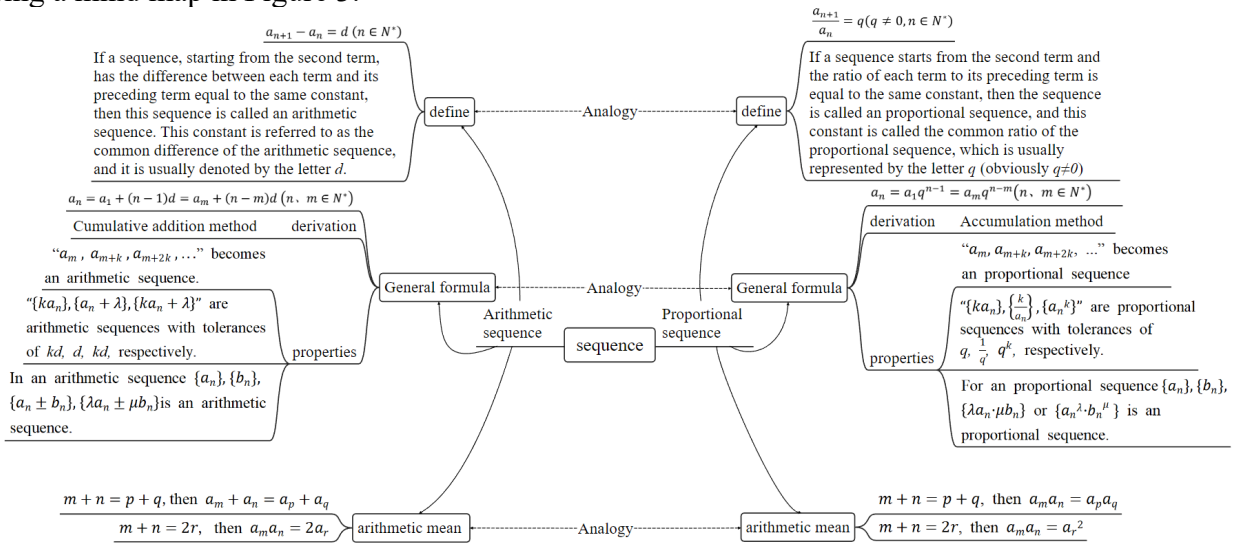

Fig. 3 Analogy of arithmetic sequences with equal differences and ratios

5.3 Reinforce and apply to understand new knowledge

Question 5: In the Cartesian coordinate system,

(1) Draw the graph of the sequence with the general term formula $a_n=2^{n-1}$.

(2) Then, draw the graph of the function $y=2^{x-1}$ in the coordinate system and observe the relationship between them.

(3) What if we change the base number to 1/2? What is the conclusion?

Design Intention: The general formula for the arithmetic sequence $\{a_n\}$ can also be written as $a_n = a_1 q^{n-1} = \frac{a_1}{q} q^n = cq^n$ (where $c = \frac{a_1}{q}$). When q is a positive number not equal to 1, $y = q^x$ is an exponential function, and $y - cq^x$ is the product of a non-zero constant and an exponential function. Therefore, from the graph, the points representing sequence $\{cq^n\}$ are all on the graph of function $y = cq^x$.

5.4 Enhance communication and summarize for system improvement

Question 6: What were the main topics discussed today? What problem-solving methods and logical thinking approaches were utilized? What issues need to be addressed?

Student Activities: Each student modified their mind map based on their understanding of mathematical concepts, reviewed lesson highlights, shared experiences, and identified reasoning methods, including inductive and deductive reasoning, as well as classification. The teacher provided feedback, clarified misconceptions, and helped connect new knowledge with prior learning, promoting a cohesive understanding. After a thorough study of geometric sequences, the final mind map was shared with the students in Figure 4.

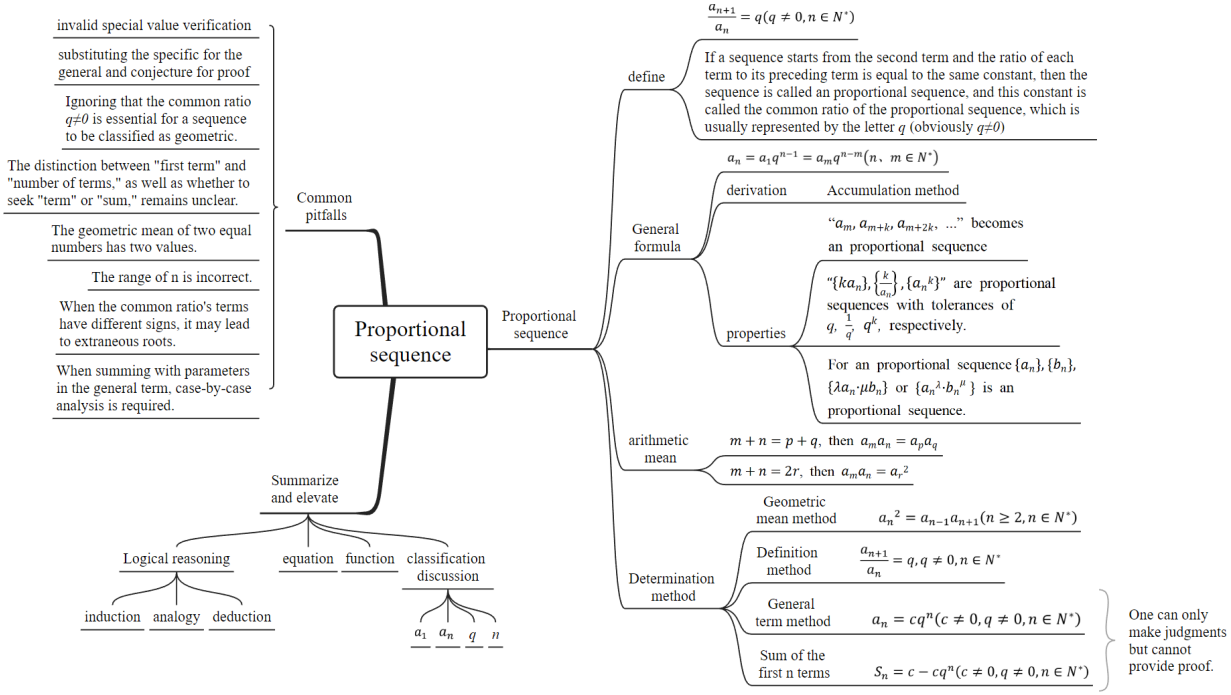

Fig. 4 Mind map of "proportional sequence"

Design Intention: The students organized and integrated fundamental knowledge of geometric sequences while applying their mathematical skills to clarify concepts and address gaps in understanding. They explored knowledge connections to strengthen their abstraction and logical reasoning abilities.

Teaching Reflection: Teachers provoke students' critical thinking with a set of questions, allowing them to explore and address these issues through dialogue and expression. This approach converts lesson content into personal understanding and strategies applicable across different subjects. Students utilize holistic thinking to synthesize information and craft mind maps, promoting knowledge dissemination and enhancing critical thought[15].

6. Summary

In summary, the demand for high school teaching quality is evolving alongside ongoing reforms

in the new curriculum standards. However, high school mathematics education still faces several optimization challenges. Therefore, frontline math teachers should prioritize fostering students' logical reasoning skills as a vital aspect of their teaching. Educators must keep pace with the times, continually updating their educational philosophies and enhancing their understanding of logical reasoning. They should create contextually relevant scenarios in their teaching, integrate thinking training and communication throughout the process, emphasize the use of mind maps, and focus on summarizing feedback to clarify concepts. This approach will cultivate students' abilities to analyze and solve problems using logical reasoning, thereby developing their comprehensive qualities and promoting holistic growth, ultimately advancing the overall educational standards.

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